Instructions To the Candidates:
1. There are 3 sections in the question paper. Write each section on separate page.
2. All Sections are compulsory.
3. Figures to the right indicate full marks.
4. Draw a well labelled diagram wherever necessary.

SECTION A

Q 1)Attempt any FIVE of the following.   [10 Marks]

i) Sketch the level curve of the function.

$$g(x, y) = \sqrt{9 - x^2 - y^2} \ \ for \ \ k = 0, 1, 2, 3.$$

ii) State the Sandwich principle.

iii) Show that following limit does not exit.

$$\lim_{(x,y) \to (0,0)} \left(\frac{xy^2}{x^2 + y^4} \right).$$

iv) Using polar co-ordinate find the following limit.

$$\lim_{(x,y) \to (0,0)} \frac{sin(x^2 + y^2)}{x^2 + y^2}.$$

v) Evaluate

$$\int_{-3}^{3} \int_{0}^{\frac{\pi}{2}} (y + y^2 cosx) dxdy.$$

vi) Find the Jocobian of the transformation. $x = uv, \ y = \frac{u}{v}$.

vii) Evaluate

$$\int_{0}^{1} \int_{x}^{2x} 2xydydx.$$

SECTION B

Q.2) Attempt any THREE of the following.   [15 marks.]

i) Use the definition of partial derivative as limits to find $f_x(x, y)$ and $f_y(x, y)$ if $f(x, y) = y^5 - 3xy$.

ii) Verify that the function $Z = ln(e^x + e^y)$ is a solution of the differential equation

$$Z_{xx}Z_{yy} - (Z_{yx})^2 = 0.$$

iii) Show that any function of the form

$$z = f(x + at) + g(x - at)$$

is a solution of the wave equation.

iv) Evaluate

$$\int\int_D xy\,dx\,dy.$$

where D is the region enclosed by the curves $y = x^2$ and $y = 3x$.

v) If $z = x^4 + x^2y$, $x = s + 2t - u$, $y = stu^2$, then find

$$\frac{\partial z}{\partial s}, \frac{\partial z}{\partial t}, \frac{\partial z}{\partial u} \quad at \quad s = 4, t = 2, u = 1.$$

SECTION C

Attempt any one of the following. [10 Marks]

i) a) State and prove Eulers theorem for homogeneous function.

b) If

$$g(s, t) = f(s^2 - t^2, t^2 - s^2)$$

and f is differentiable then show that g satisfies the equation

$$t.\frac{\partial g}{\partial s} + s.\frac{\partial g}{\partial t} = 0$$

ii. a)A rectangular box without a lid is to be made from $12m^2$ of cardboard. Find the maximum volume of such a box.(use langrange multiplier method).

b) Evaluate double integral

$$\int\int_R \frac{xy^2}{x^2 + 1}dx\,dy$$

where the region R is given as

$$R = \{(x, y) : 0 \le x \le 1, -3 \le y \le 3\}.$$

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