



Progressive Education Society's  
Modern college of Arts, Science & Commerce,  
Ganeshkhind, Pune 16  
End Semester Examination October 2023  
Faculty: Science and Technology

Program: B.Sc. Code(BscGen03) Semester III Set B  
Program(Specific): B.Sc. General Course Type:Core  
Class:S.Y.BSc.(Regular) Max. Marks:35  
Name of the Course: Calculus of several variable  
Course Code: 23-MT-231  
Paper No: I Time: 2 Hours

**Instructions To the Candidates:**

1. There are 3 sections in the question paper. Write each section on separate page.
2. All Sections are compulsory.
3. Figures to the right indicate full marks.
4. Draw a well labelled diagram wherever necessary.

*SECTION A*

**Q 1) Attempt any FIVE of the following.** [10 Marks]

- i) Sketch the level curve of the function.

$$g(x, y) = \sqrt{9 - x^2 - y^2} \text{ for } k = 0, 1, 2, 3.$$

- ii) State the Sandwich principle.

- iii) Show that following limit does not exit.

$$\lim_{(x,y) \rightarrow (0,0)} \left( \frac{xy^2}{x^2 + y^4} \right).$$

- iv) Using polar co-ordinate find the following limit.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{\sin(x^2 + y^2)}{x^2 + y^2}.$$

- v) Evaluate

$$\int_{-3}^3 \int_0^{\frac{\pi}{2}} (y + y^2 \cos x) dx dy.$$

- vi) Find the Jacobian of the transformation.  $x = uv$ ,  $y = \frac{u}{v}$ .

- vii) Evaluate

$$\int_0^1 \int_x^{2x} 2xy dy dx.$$

*SECTION B*

**Q.2) Attempt any THREE of the following.** [15 marks.]

i) Use the definition of partial derivative as limits to find  $f_x(x, y)$  and  $f_y(x, y)$  if  $f(x, y) = y^5 - 3xy$ .

ii) Verify that the function  $Z = \ln(e^x + e^y)$  is a solution of the differential equation

$$Z_{xx}Z_{yy} - (Z_{yx})^2 = 0.$$

iii) Show that any function of the form

$$z = f(x + at) + g(x - at)$$

is a solution of the wave equation.

iv) Evaluate

$$\int \int_D xy dx dy.$$

where D is the region enclosed by the curves  $y = x^2$  and  $y = 3x$ .

v) If  $z = x^4 + x^2y$ ,  $x = s + 2t - u$ ,  $y = stu^2$ , then find

$$\frac{\partial z}{\partial s}, \frac{\partial z}{\partial t}, \frac{\partial z}{\partial u} \text{ at } s = 4, t = 2, u = 1.$$

### SECTION C

**Attempt any one of the following.**

[10 Marks]

i) a) State and prove Eulers theorem for homogeneous function.

b) If

$$g(s, t) = f(s^2 - t^2, t^2 - s^2)$$

and f is differentiable then show that g satisfies the equation

$$t \cdot \frac{\partial g}{\partial s} + s \cdot \frac{\partial g}{\partial t} = 0$$

ii. a) A rectangular box without a lid is to be made from  $12m^2$  of cardboard. Find the maximum volume of such a box. (use langrange multiplier method).

b) Evaluate double integral

$$\int \int_R \frac{xy^2}{x^2 + 1} dx dy$$

where the region R is given as

$$R = \{(x, y) : 0 \leq x \leq 1, -3 \leq y \leq 3\}.$$

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